

# **Strangeness kinetics and energy dependence of hadron productions**

Kinetic equation and conservation laws

Chemical equilibration from SIS to RHIC

Strangeness production energy dependence

Maximum relative strangeness content in heavy ion collisions

Work with:

P. Braun-Munzinger

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J. Stachel

# Statistical-Thermal Models

- At freeze-out: the available particle phase space is occupied according to statistical laws  
-> thermal distribution
- Chemical freeze-out: particle abundances are frozen in
- Thermal freeze-out: particle spectra are frozen in

Test of equilibration required to specify:

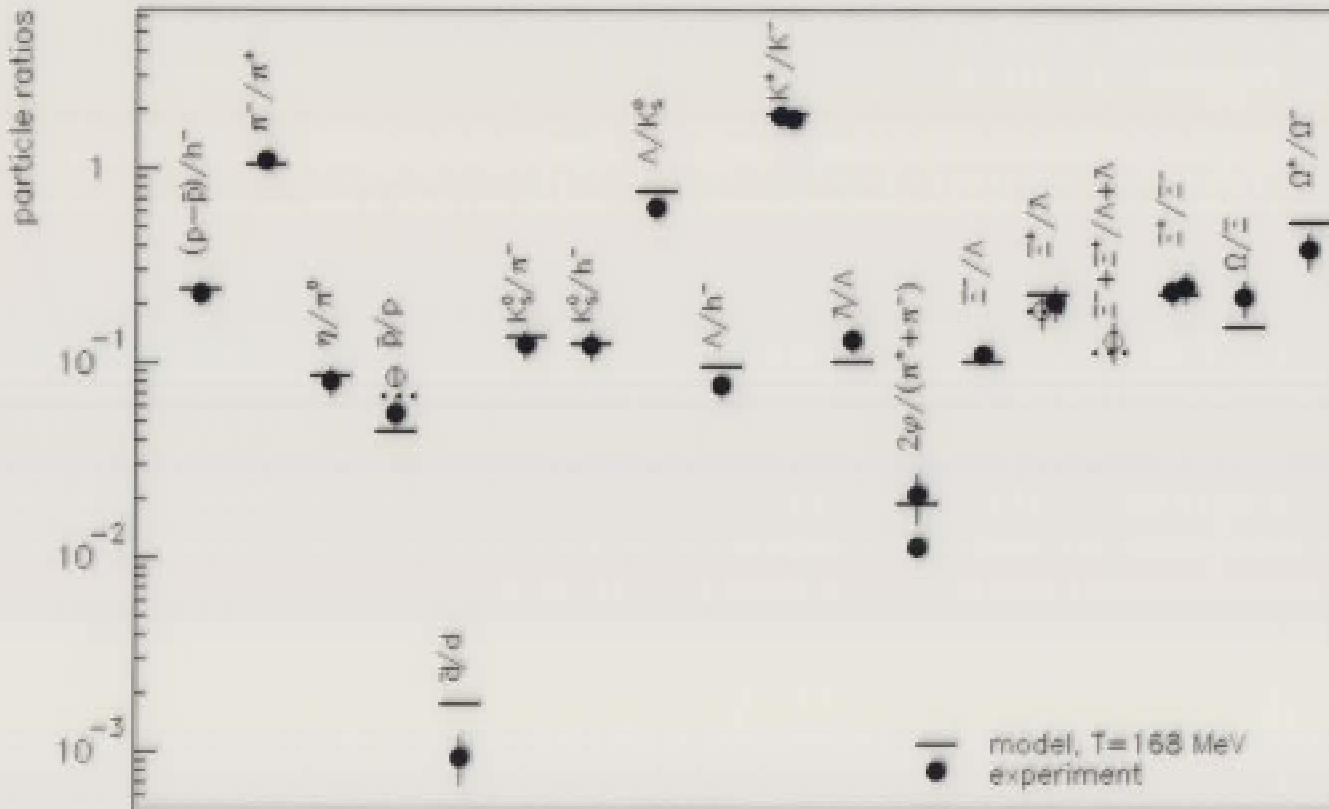
- i) level of observation: ( multiplicities, spectra, correlations,...)
- ii) Statistical operator:

$$Z^{GC}(T, \vec{\mu}, V) = \text{Tr}[e^{-\beta(H - \mu_B B - \mu_S S - \mu_Q Q)}]$$

# Test of chemical equilibrium for Pb-Pb at SPS

P. Braun-Munzinger, I. Heppe, J. Stachel

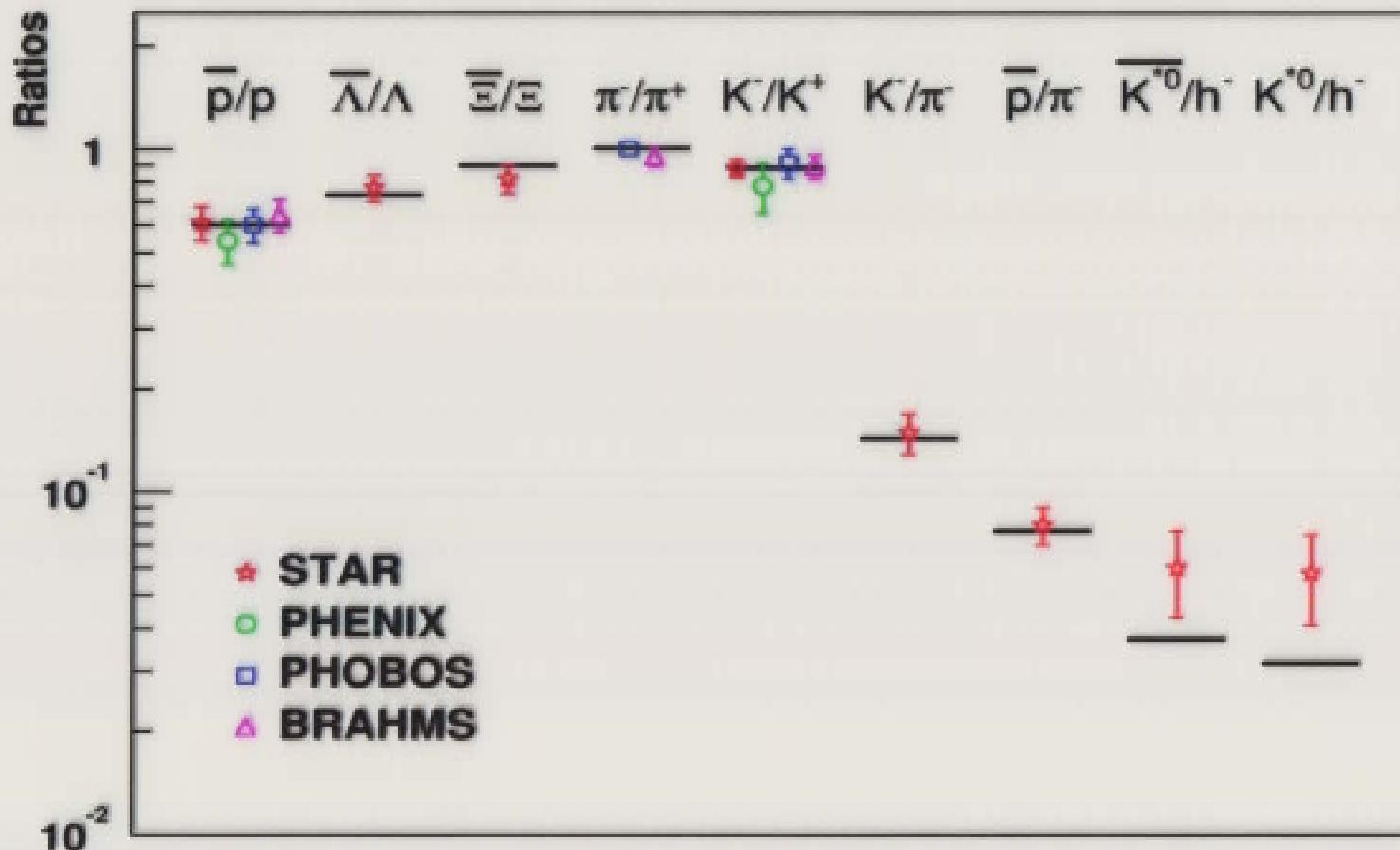
$$\varepsilon \approx 0.6 \text{GeV / fm}^3, \rho_B \approx 0.16 / \text{fm}^3$$



# Statistical description of RHIC data

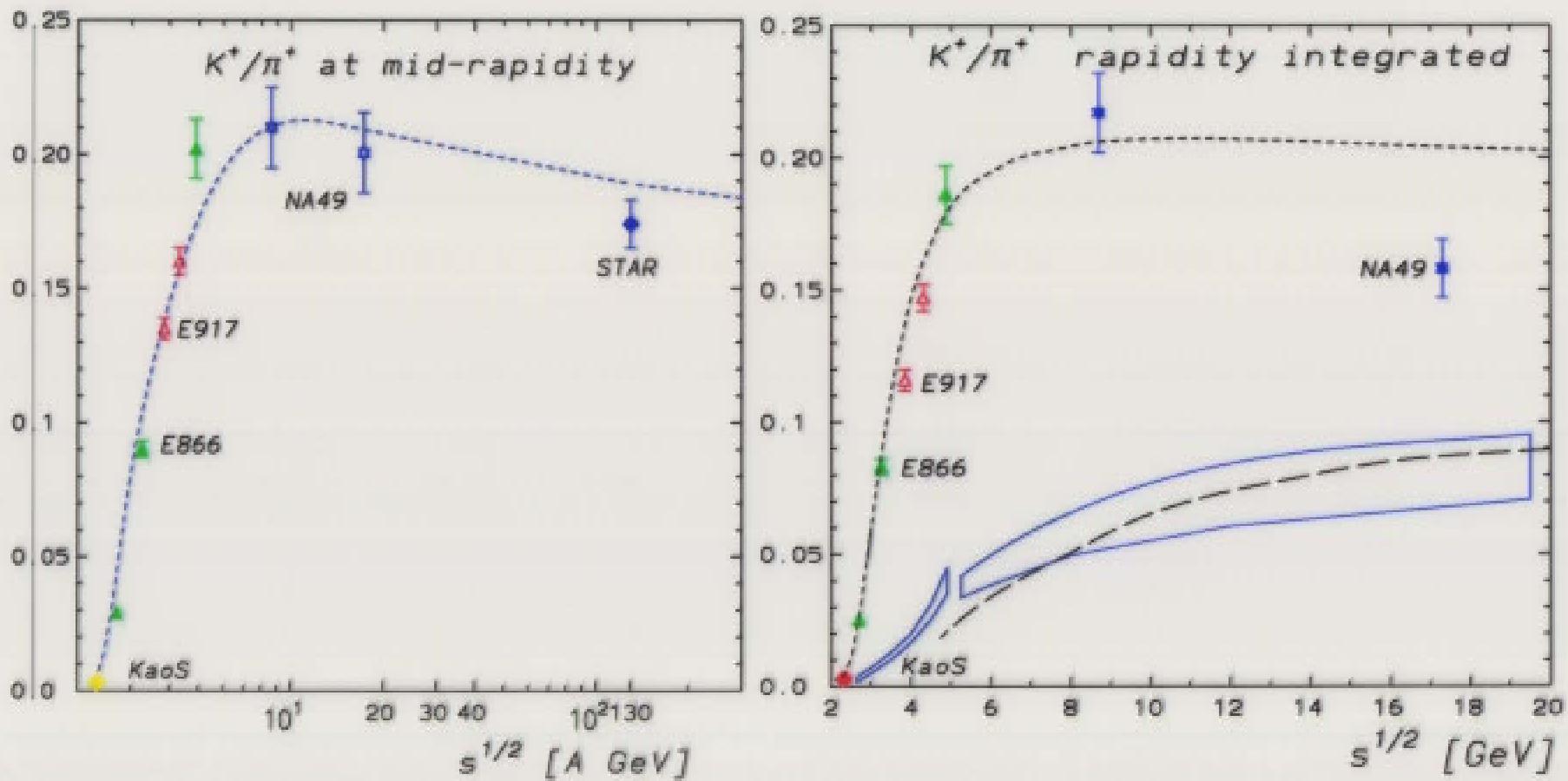
P. Braun-Munzinger, D. Magestro, J. Stachel, K.R.

$$T = 175 \pm 7 \text{ MeV} \quad \mu_B = 51 \pm 5 \text{ MeV}$$



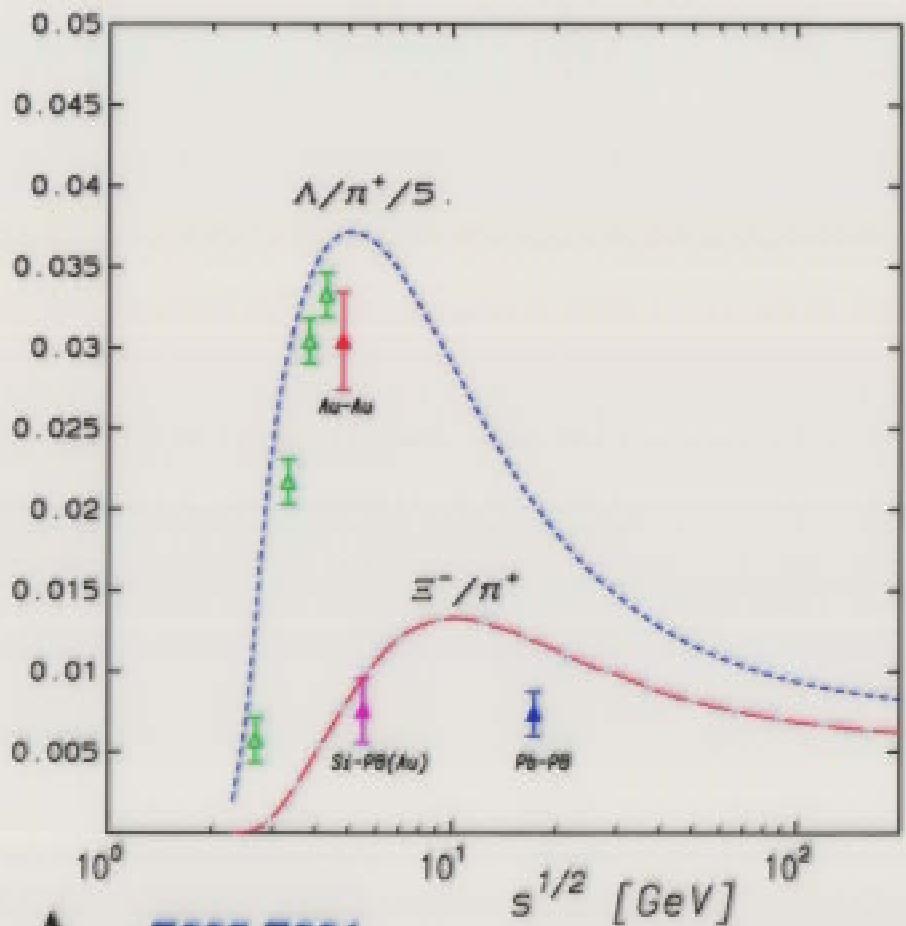
consistent with: Nu. Xu & Kaneta; Broniowski & Florkowski ; F. Becattini

# Mid-rapidity and 4pi $K^+/\pi^+$ data from SIS to RHIC



# $\Lambda$ and $\Xi^-$ energy dependence from SIS $\rightarrow$ RHIC

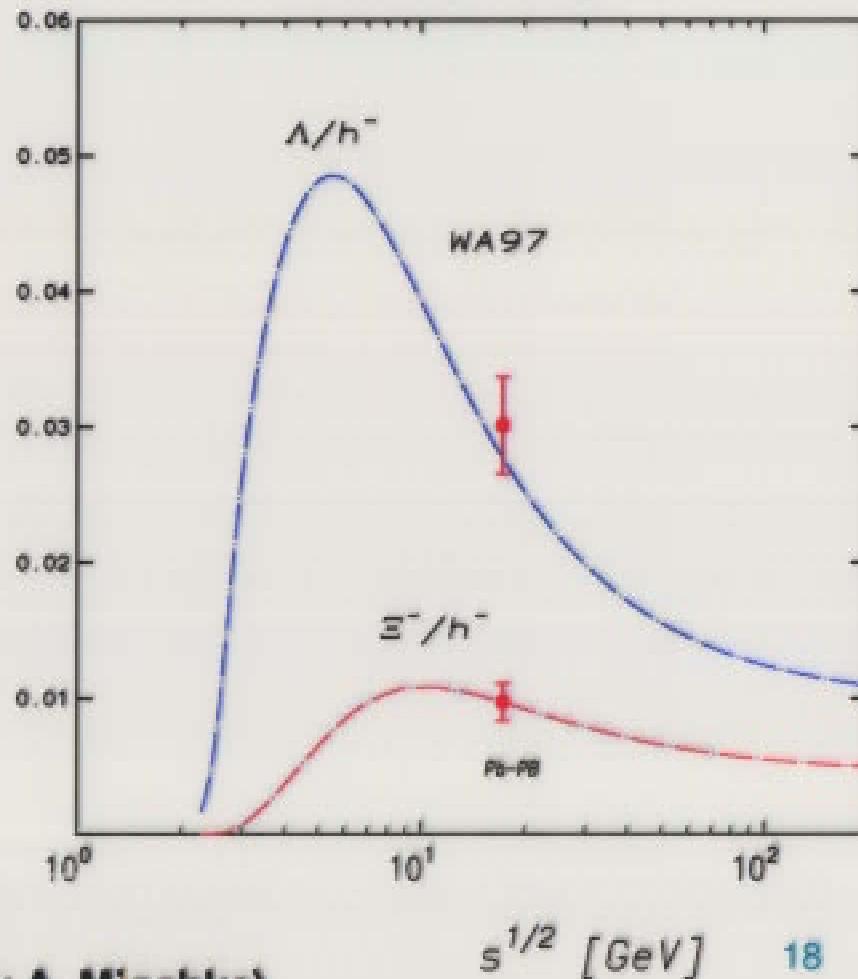
(4  $\pi$  data)



$\Lambda$  - E895,E891  
pi - E866,8917

( $\Lambda/\pi$  ratio analysis by A. Mischke)

midrapidity



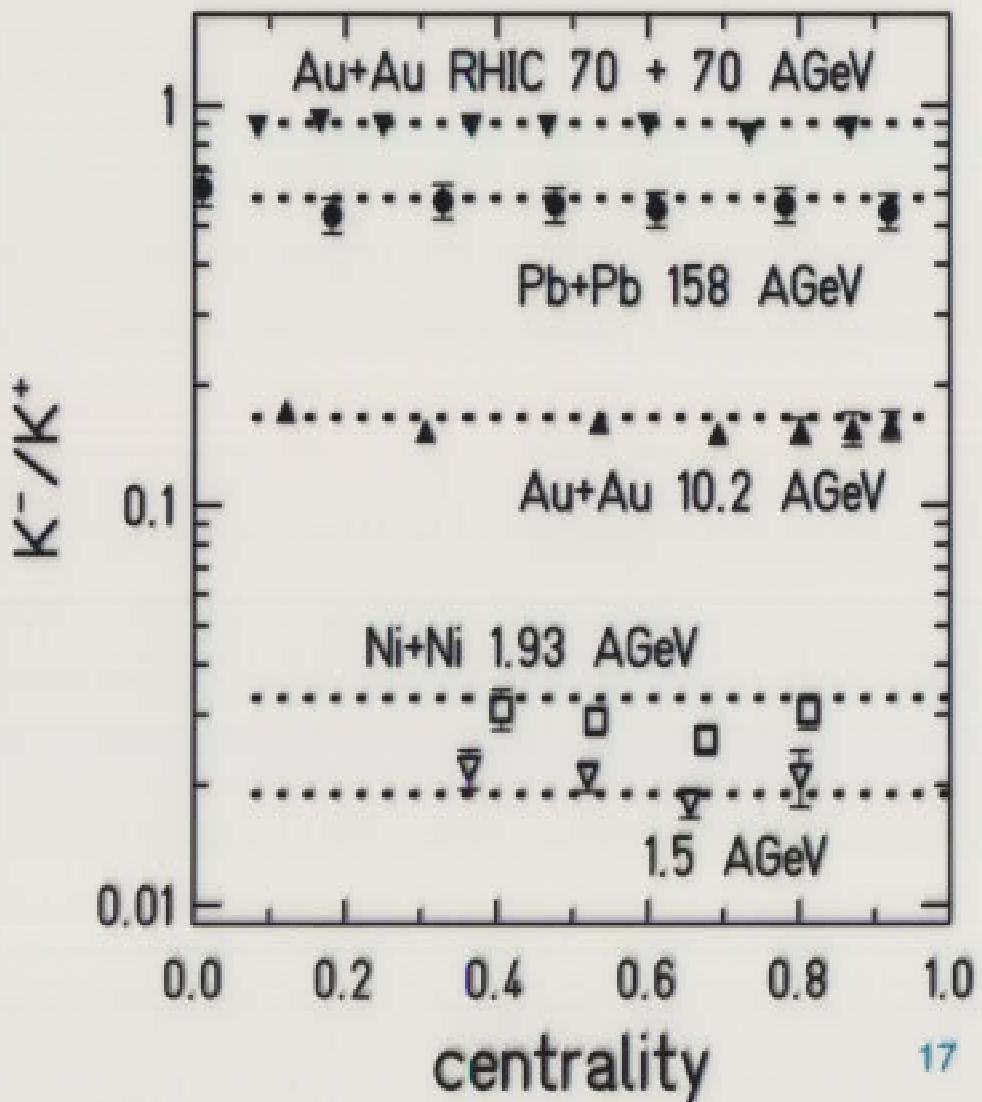
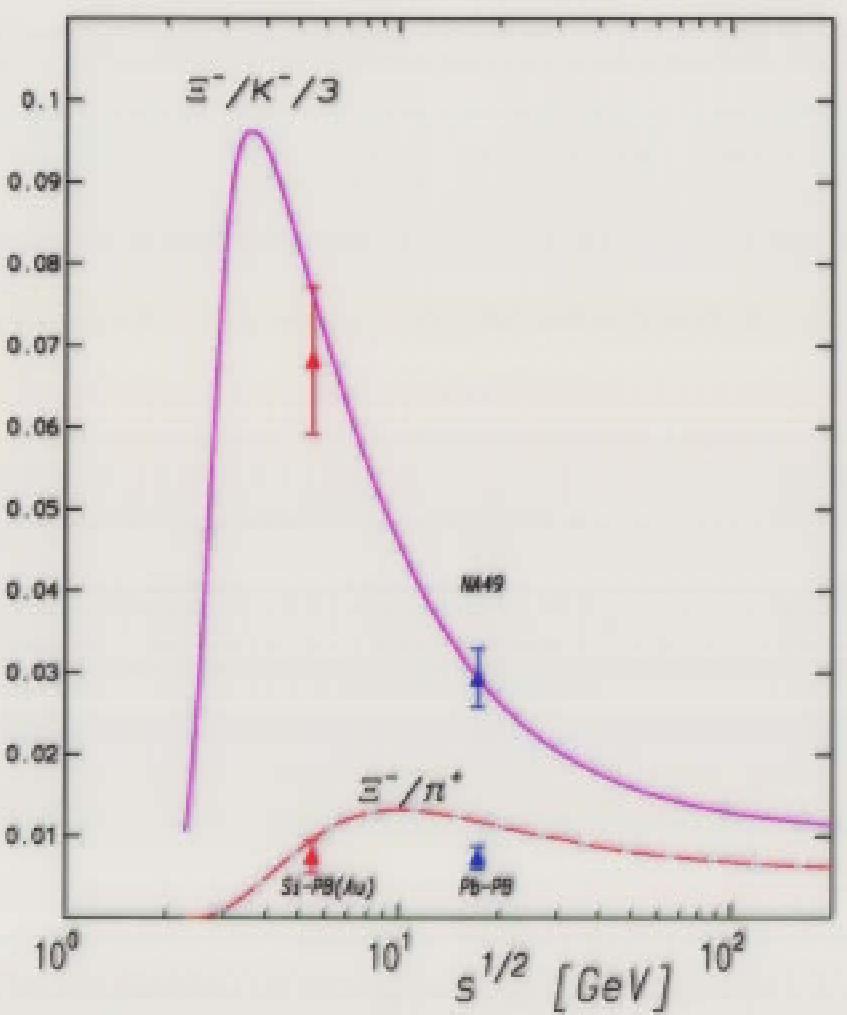
$s^{1/2}$  [GeV] 18

# Particle production – energy dependence

E810,E802,NA49

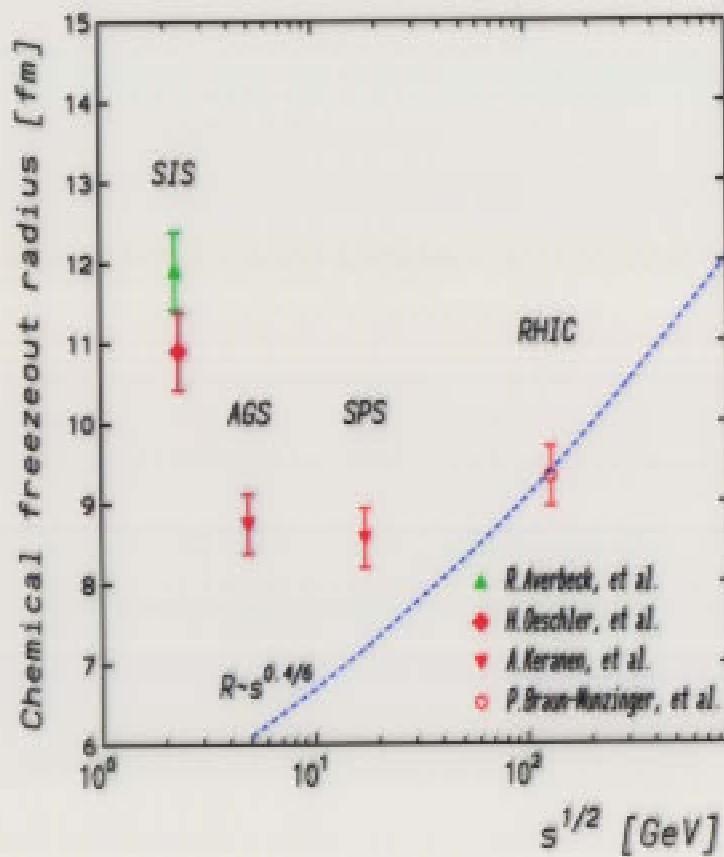
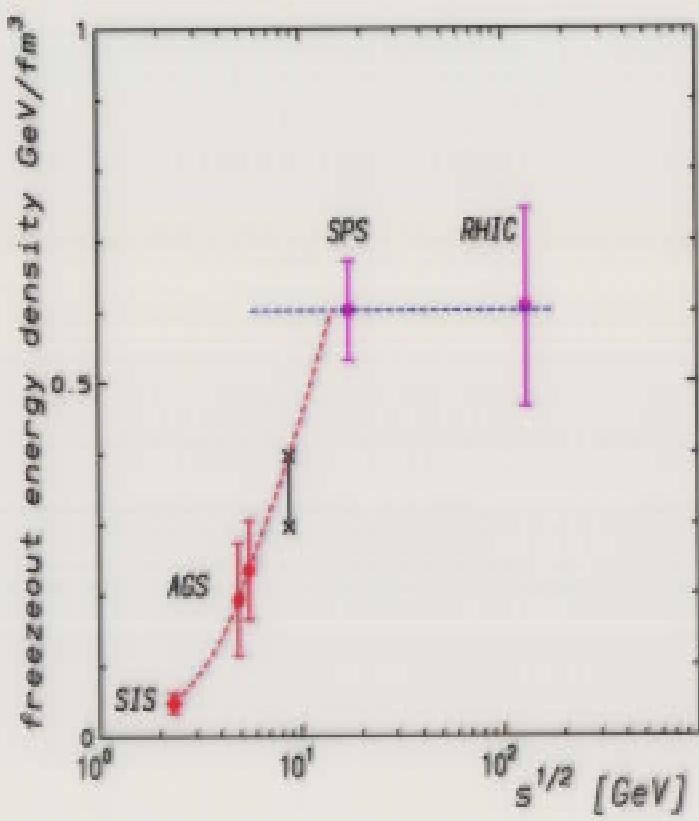
<=data from=>

STAR,NA49,E802,FOPI,KaOS

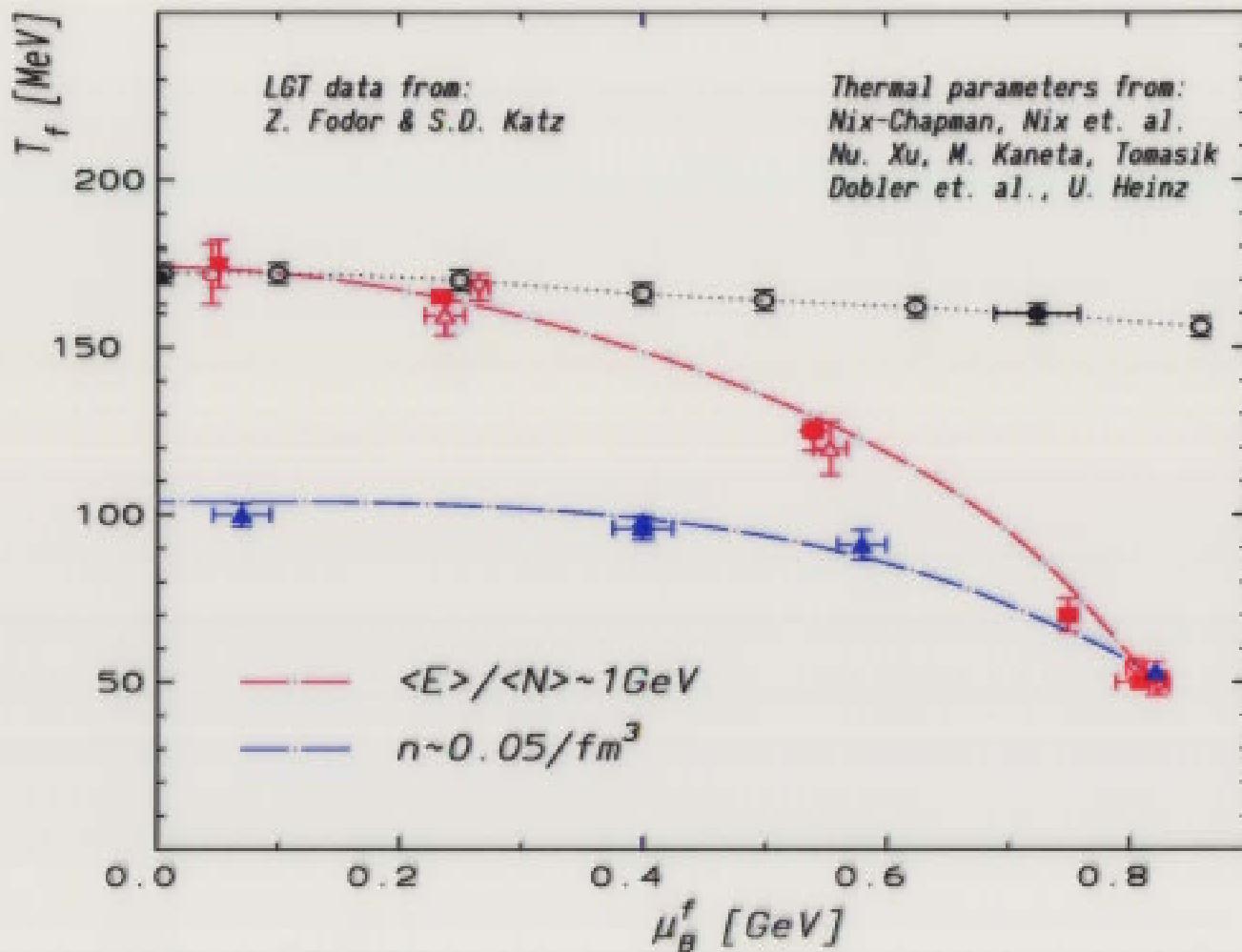


# Thermal parameters at chemical freezeout

## Energy Density      Radius



## Thermal, chemical and “critical” curves



First attempts to extract “critical parameters” from LGT at finite baryon density: based on the large volume behavior of the Lee-Yang zeros (Fodor & Katz)

## Solution: convert to differential equation for the generating function

$$g(x, t) = \sum_{N=0}^{\infty} x^N P_N(t)$$

$$\frac{\partial g(x,t)}{\partial t} = \frac{L}{V_0} (1-x) \left( x \frac{\partial^2 g}{\partial x^2} + \frac{\partial g}{\partial x} - \mathcal{E} g \right) \quad \mathcal{E} \equiv \frac{G}{L} < N_\pi >^2$$

Equilibrium solution

$$g_{eq}(x) = \frac{I_0(2\sqrt{x\varepsilon})}{I_0(2\sqrt{\varepsilon})} \implies P_N^{eq} = \frac{\varepsilon^N}{I_0(2\sqrt{\varepsilon})(N!)^2}$$

Kaon Multiplicity

$$< N_K >^{eq} = \left[ \frac{\partial g(x)}{\partial x} \right]_{x=1} \implies < N_K >^{eq} = \sqrt{\varepsilon} \frac{I_1(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})}$$

## Strangeness multiplicity equilibrium solution

$$\langle N_K \rangle^{eq} = \sqrt{\varepsilon} \frac{I_1(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})} \quad \text{with} \quad \sqrt{\varepsilon} \equiv \frac{Vd}{2\pi^2} m_K^2 T K_2\left(\frac{m_K}{T}\right)$$

↓

$$\langle N_K \rangle^{GC}$$

$$\langle N_K \rangle^{eq} = \langle N_K \rangle^{GC} \frac{I_1(2\langle N_K \rangle^{GC})}{I_0(2\langle N_K \rangle^{GC})}$$

# (Multi) Strange Particle Multiplicities

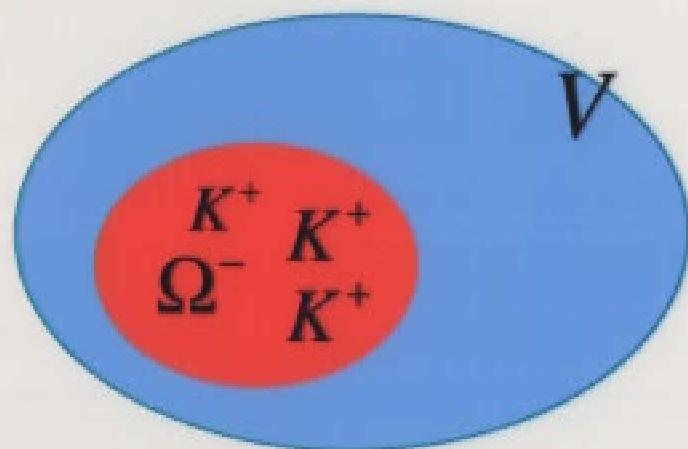
Hamieh, Tounsi, et al..

Consider : baryon free and charge neutral system  
of volume  $V$  and temperature  $T$

Impose : strangeness neutrality condition

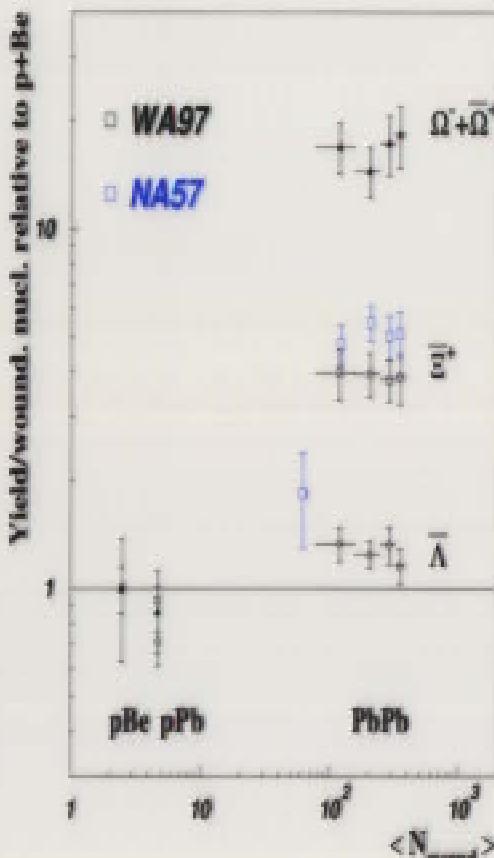
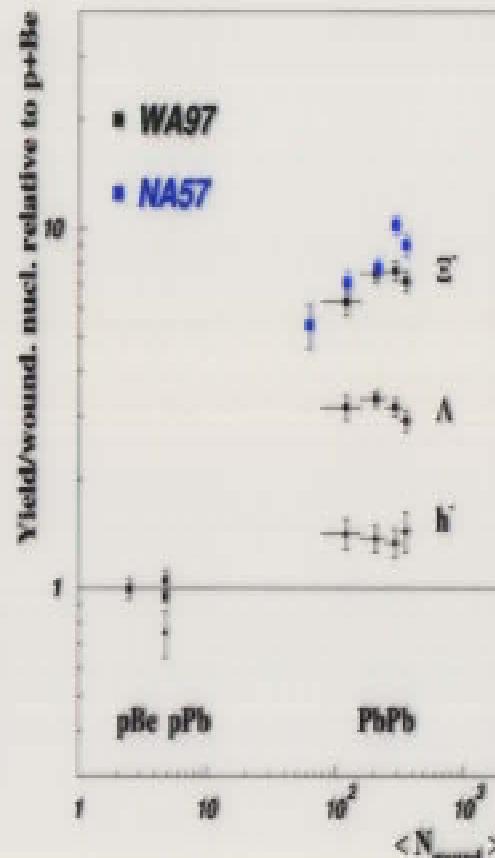
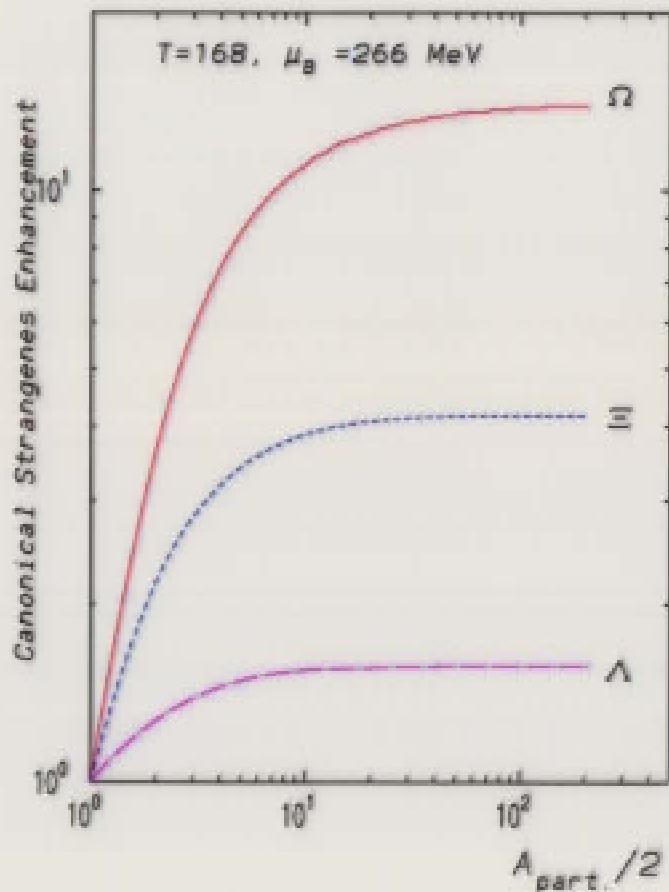
$$\frac{\langle N_s \rangle}{V} \approx w_s \frac{I_s(2Vw_{S=1})}{I_0(2Vw_{S=1})}$$

$$w_s \sim \int d^3 p e^{-\beta E_s}$$



$$\frac{\langle N_s \rangle^c}{V} \approx \begin{cases} Vw_{S=1} \gg 1: & w_s \\ Vw_{S=1} \ll 1: & w_s \{ [Vw_{S=1}]^{|S|} + \dots \} \end{cases}$$

# Statistical Model – Centrality Dependence approaching asymptotic grand-canonical limit

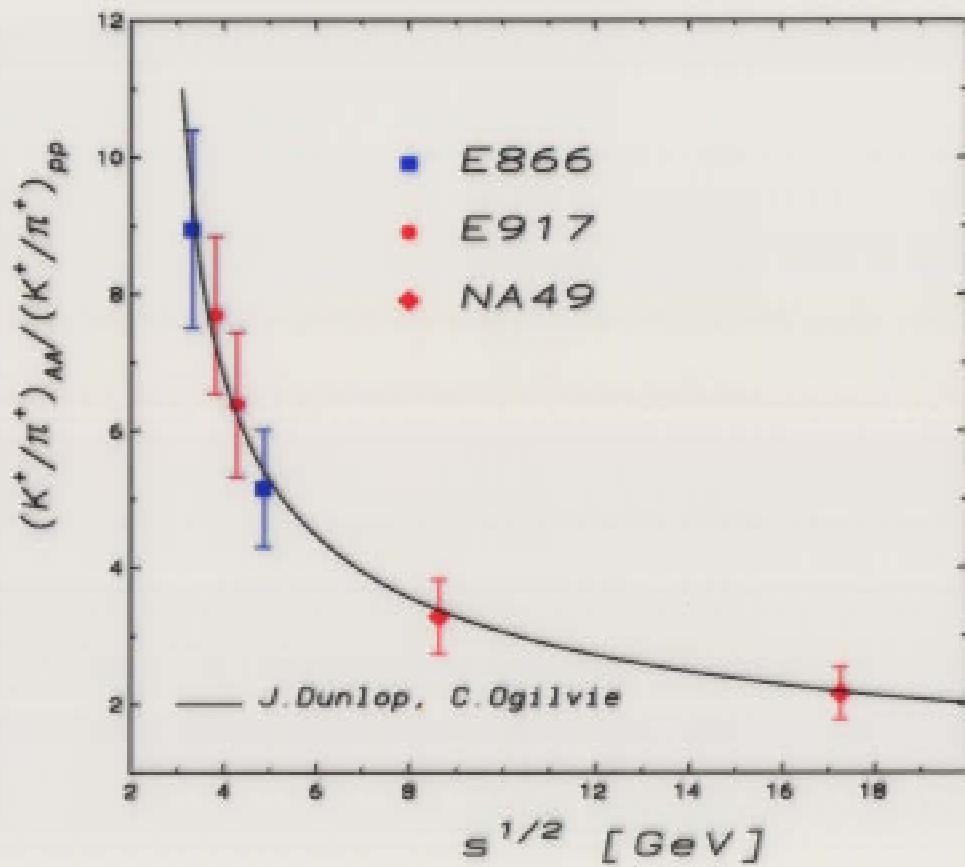
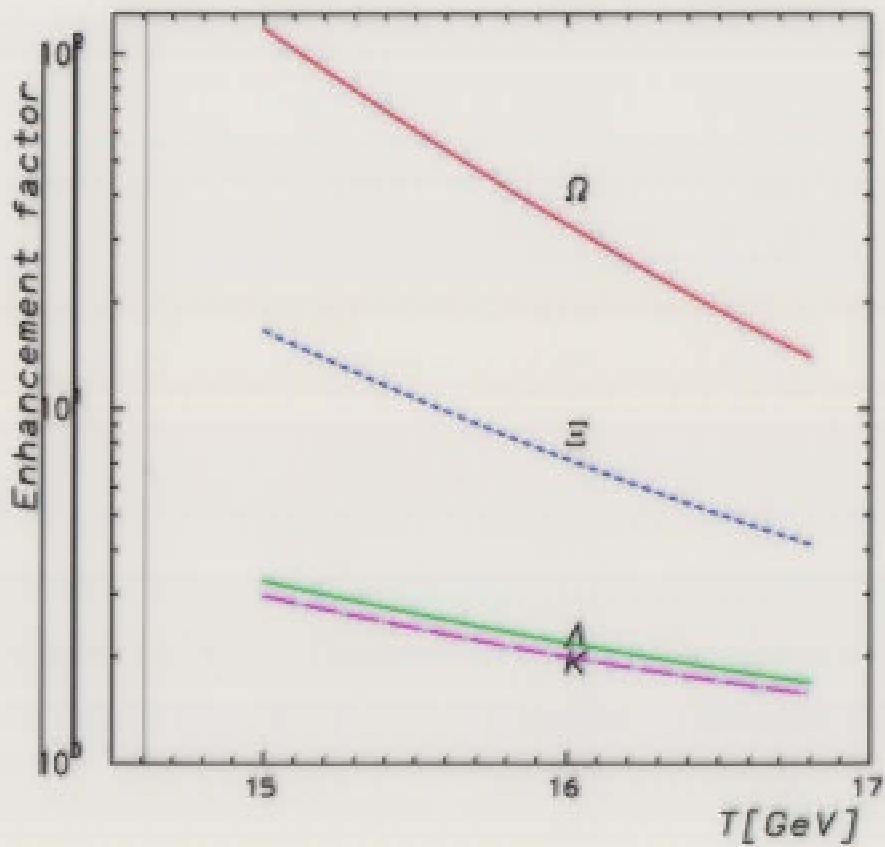


pattern as observed  
by WA97

# Strangeness enhancement

pp  $\Rightarrow$  AA

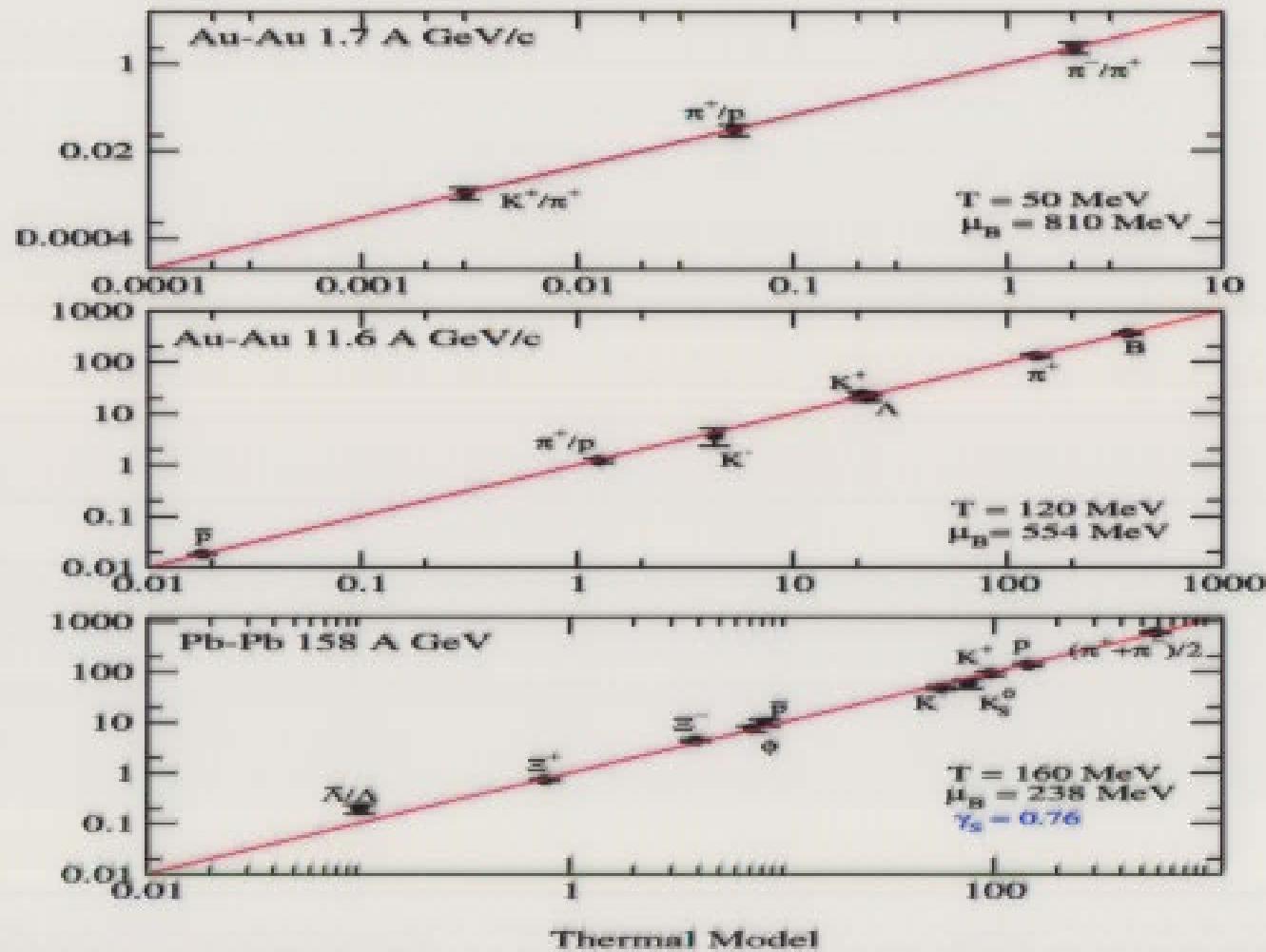
## energy dependence



Strangeness enhancement  
larger for lower energy

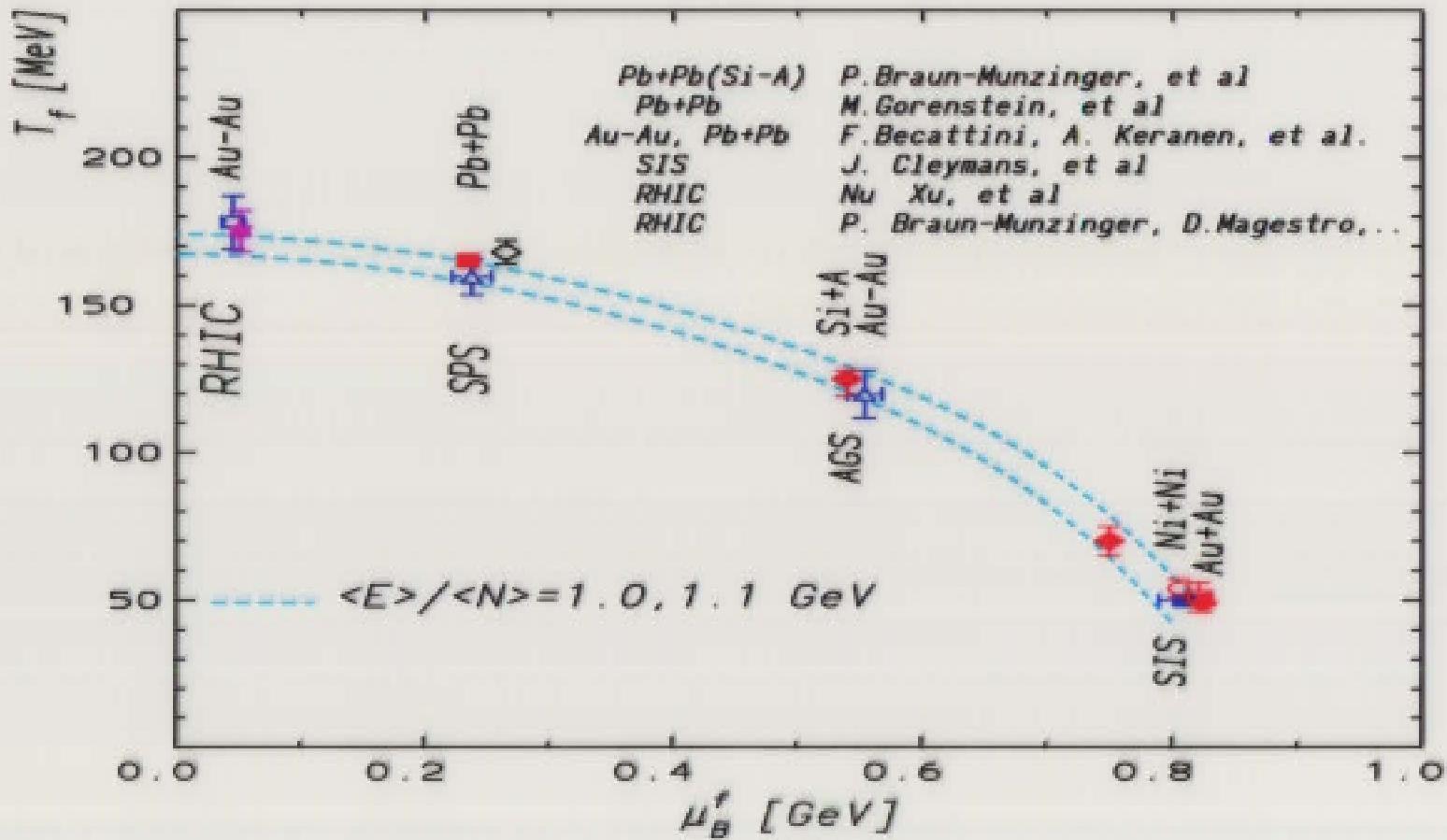
# Chemical Equilibrium SIS - AGS - SPS

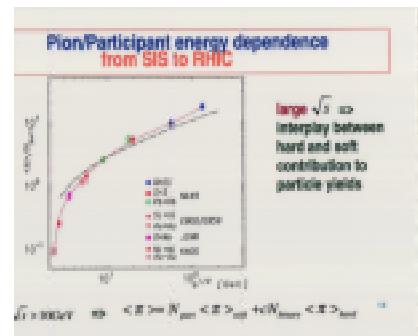
F. Becattini, J. Cleymans, A. Keranen, H. Oeschler, E. Suhonen, et al..



# Unified freeze-out curve from SIS => RHIC

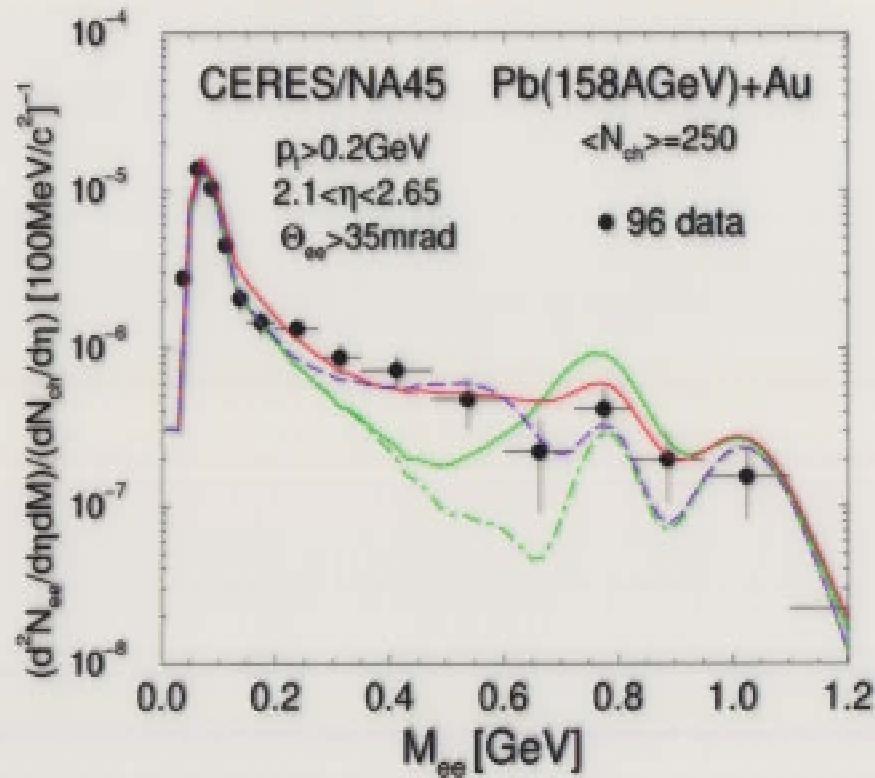
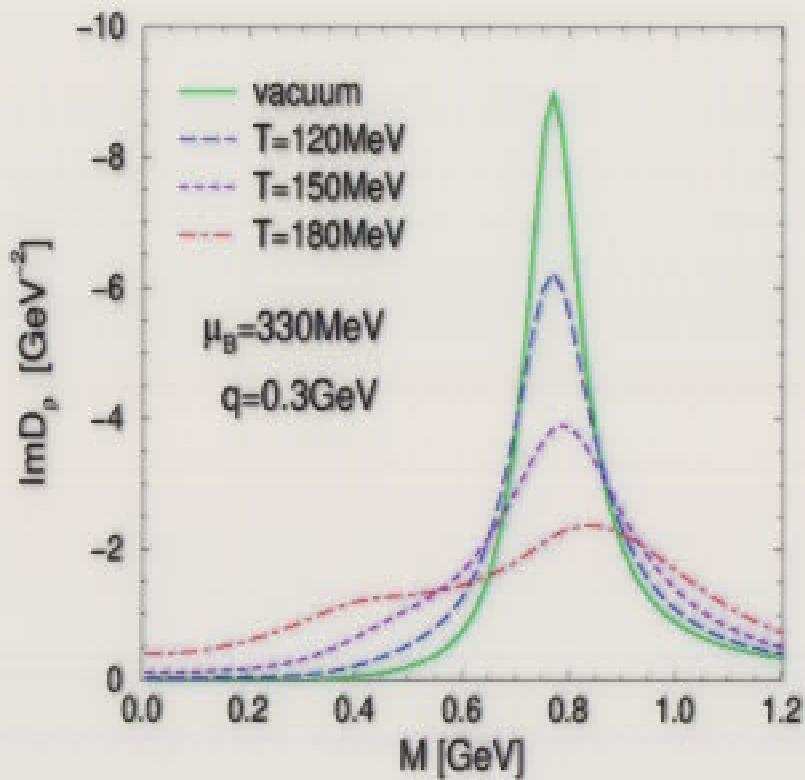
J. Cleymans, K.R





# In medium effects –resonace broadening

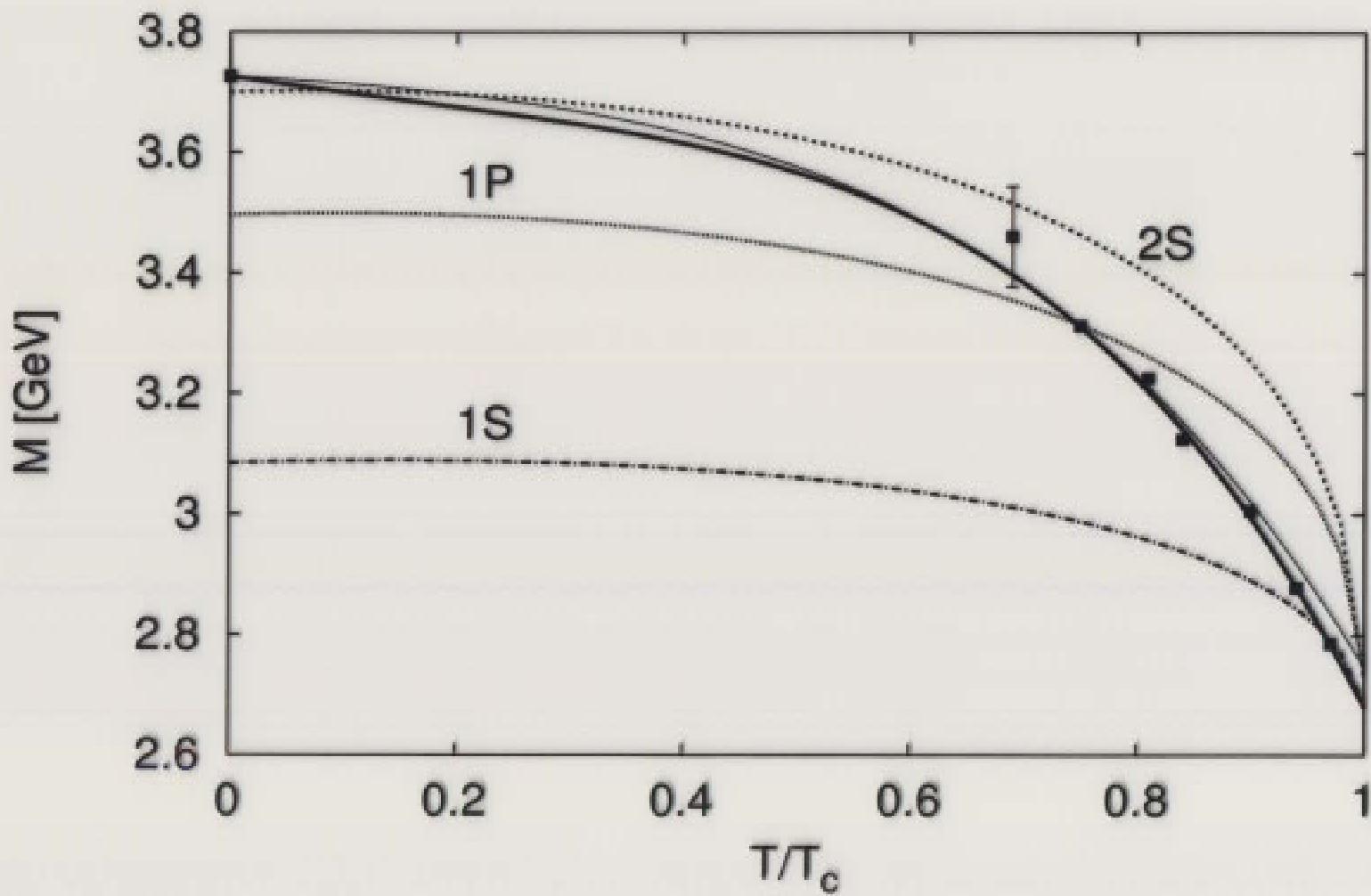
R. Rapp & J. Wambach



In medium effects => required to explain dilepton data however are not included in statistical analysis of particle yields

# Charm in-medium LGT results

S.Digal, P. Petreczky and H. Satz



# Strangeness suppression – kinetic approach

C.M. Ko, V. Koch, Z. Lin, M. Stephanov, Xin-Nian Wang, K.R

Example:



Rate equation:

$$\frac{d \langle N_K \rangle}{dt} = \frac{G}{V} \langle N_\pi \rangle^2 - \frac{L}{V} \langle N_K \rangle^2$$

$$\langle N_K^2 \rangle = \langle N_K \rangle^2 + \langle \delta N_K^2 \rangle$$



$$\langle N_K \rangle$$

Size of fluctuations



Equilibrium limit

$$\langle N_K \rangle \ggg 1$$

$$\langle N_K \rangle \ll 1$$

$$n_{K^+} \approx m_{K^+}^2 T K_2 \left( \frac{m_{K^+}}{T} \right)$$

$$n_{K^+} \approx m_{K^+}^2 T K_2 \left( \frac{m_{K^+}}{T} \right) \times$$

$$V m_{K^-}^2 T K_2 \left( \frac{m_{K^-}}{T} \right)$$

# Strangeness kinetic – general approach

$P_N(t)$  probability of finding  $N$  pairs of  $K^+K^-$



Transition probability from  $N \rightarrow N+1$ :

$$\frac{G}{V} < N_\pi >^2$$

Transition probability from  $N \rightarrow N-1$ :

$$\frac{L}{V} N^{-2}$$

$$\frac{dP_N}{dt} = \frac{G}{V} < N_\pi >^2 P_{N-1} + \frac{L}{V} (N+1)^2 P_{N+1} -$$

$$\frac{G}{V} < N_\pi >^2 P_N - \frac{L}{V} N P_N -$$

$$< N_K > = \sum_N N P_N$$